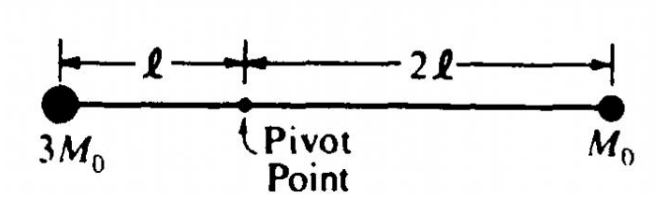
Lesson 7: Moment of inertia and Angular momentum (C)

Q1. A uniform stick has length L. The moment of inertia about the center of the stick is I_o. A particle of mass M is attached to one end of the stick. The moment of inertia of the combined system about the center of the stick is

(A) $I_0 + \frac{1}{4}ML^2$ (B) $I_0 + \frac{1}{2}ML^2$ (C) $I_0 + \frac{3}{4}ML^2$ (D) $I_0 + ML^2$

Q2. A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown in the figure. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude



(A) $\frac{g}{7l}$ (B) $\frac{g}{5l}$ (C) $\frac{g}{4l}$ (D) $\frac{5g}{7l}$

Q3. A turntable initially at rest is set in motion with a constant angular acceleration α. What is the angular velocity of the turntable after it has made one complete revolution?

(A) $\sqrt{2\alpha}$ (B) $\sqrt{2\pi\alpha}$ (C) $\sqrt{4\pi\alpha}$ (D) $4\pi\alpha$

For Q4, Q5, Q6: A wheel with rotational inertia I is mounted on a fixed and frictionless axle. The angular speed ω of the wheel is increased from zero to ω_f in a time interval T.

Q4. What is the average net torque on the wheel during this time interval?

(A) $\frac{\omega_f}{T}$ (B) $\frac{I\omega_f^2}{T}$ (C) $\frac{I\omega_f}{2T^2}$ (D) $\frac{I\omega_f}{T}$

Q5. What is the amount of work done by the net torque to the wheel during this time interval?

(A) ½I ω_f^2 (B) I ω_f^2 (C) I ω_f T (D) I ω_f^2/T

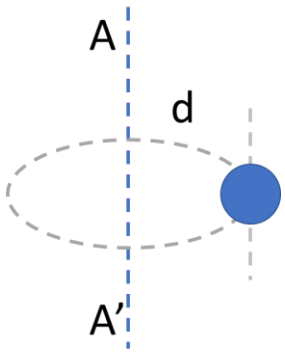
Q6. What is the average power input to the wheel during this time interval (hint: power = work / time)?

(A) $\frac{I\omega_f}{2T}$ (B) $\frac{I\omega_f^2}{2T}$ (C) $\frac{I\omega_f^2}{2T^2}$ (D) $\frac{I^2\omega_f}{2T^2}$

Q7. Orbiting rotational inertia

A sphere has a mass of m and a radius of R. It orbits around the AA' axis. The distance between the axis and the sphere center is d. What is the rotational inertia of this sphere about axis AA'?

$2mR^2/5$
$m(d+R)^2$
md^2
mR^2
$2mR^2/5 + md^2$



Q8. A wheel with 3 spokes

A wheel is made of a thin hoop and three thin spokes. The hoop's mass is m and the radius is R. Each spoke has a mass of m/6 and a length of R.

What is its rotational inertia about the axis going through its center and perpendicular to the plane of the hoop?

- $3mR^2/2$
- $19mR^2/18$
- $7mR^2/6$
- mR^2
- $4mR^2$

What is its total mass?

- 7m/6
- 3m/2
- 3m
- 2m
- 4m

When it rolls without slipping, what is the ratio between its rotational KE and linear KE?

- 7/9
- 7/6
- 3/2
- 9/7
- 6/7
- 2/3

It rolls down from rest on an inclined plane. When its center of mass drops by a height of h, how much potential energy is converted into kinetic energy?

- 3mgh
- mgh/2
- 4mgh/3
- 3mgh/2
- mgh

What is the speed of the center of mass at that moment?

A. $\frac{9}{16}\sqrt{gh}$

B. $\frac{7}{9}\sqrt{gh}$

C. $\frac{3}{4}\sqrt{2gh}$

D. $\frac{3}{4}\sqrt{gh}$

E. $\sqrt{12gh}/\sqrt{13}$

Q9. Most relevant equations for rotation applications

Which of the following two equations are the most relevant ones to be used **together** to explain the following phenomena or answer the following question **directly**?

Why do discus throwers turn a couple circles to throw it?

A. $\tau = F\,r$
B. $\tau\,\Delta\theta = \text{work}$
C. $\tau\,t = L_f - L_i$
D. $F\,t = \Delta\,p$
E. $\Delta\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2$
F. $\text{Work} = KE_f - KE_i$

Why do ballet dancers tiptoe while spinning?

A. $F\,t = \Delta\,p$
B. $\Delta\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2$
C. $\text{Work} = KE_f - KE_i$
D. $\tau = F\,r$
E. $\tau\,t = L_f - L_i$

How do figure skaters control their spinning speeds?

A. $\tau = I\,\alpha$
B. $I_f\,\omega_f = I_i\,\omega_i$
C. $p_f = p_i$
D. $KE_f = KE_i$
E. $I = \text{sum of } m\,r^2$

Q10. Rotation stage video observation
Rotation stage video observation
Observe the video in the lecture and answer the following questions. Professor Man is standing on a rotational stage in the video.
How does the moment of inertia change when Professor Man extends her arms?
(increase, decrease, stay unchanged, can't be determined)
How does the rotational speed change when Professor Man extends her arms?
(increase, decrease, stay unchanged, can't be determined)
How does the moment of inertia change when Professor Man's arms are moved closer to the body?
(increase, decrease, stay unchanged, can't be determined)
How does the rotational speed change?
(increase, decrease, stay unchanged, can't be determined)

M7C Essay:
Q11. Under what condition the angular momentum of an object or a system is conserved?
Can you think of examples of angular momentum conservation?
Why do planets keep orbiting the Sun for billions of years, while the Sun is applying a huge gravitational force on the planets? Does this force change the angular momentum of the planet?
Why do ballet dancers tiptoe while spinning?
Why can a spinning top continue for a long while? Why does it have a pointy bottom?

Q12. Recall the video of Professor Man standing on the rotating stage.
How does she control her rotational speed?
How to explain what you observe here using what you have learned in this lecture?
Hint: Consider Professor Man and the rotating stage as a whole.
What is the total torque received on the system, when friction is negligible?
Professor Man is holding something heavy in her hands. Why? What are they for?
What else do you notice or find?

Q13. Why do discus throwers turn a few circles to throw it? How can it help to throw the discus further?

Q14. Why are wheels round? What is the advantage of circular wheels? Hint: how does the trace of the center of the wheel look like?

Q15. What surprised, impressed, or interested you in this lecture? What made you laugh or think? What mistake did you avoid or correct? What confusion did you clarify in this lesson?